On the Schwinger limit attainability with extreme power lasers

Stepan S. Bulanov,[1,*] Timur Zh. Esirkepov,[2] Alexander G. R. Thomas,[1] James K. Koga,[2] and Sergei V. Bulanov[2,†]

[1] *University of Michigan, Centre of Ultrafast Optical Sciences, Ann Arbor, MI 48109, USA*

[2] *Kansai Photon Science Institute, JAEA, Kizugawa, Kyoto 619-0215, Japan*

High intensity colliding laser pulses can create abundant electron-positron pair plasma [A. R. Bell and J. G. Kirk, Phys. Rev. Lett. **101**, 200403 (2008)], which can scatter the incoming electromagnetic waves. This process can prevent reaching the critical field of Quantum Electrodynamics at which vacuum breakdown and polarization occur. Considering the pairs are seeded by the Schwinger mechanizm, it is shown that the effects of radiation friction and the electron-positron avalanche development in vacuum depend on the electromagnetic wave polarization. For circularly polarized colliding pulses, which force the electrons to move in circles, these effects dominate not only the particle motion but also the evolution of the pulses. While for linearly polarized pulses, where the electrons (positrons) oscillate along the electric field, these effects are not as strong. There is an apparent analogy of these cases with circular and linear electron accelerators with the corresponding constraining and reduced roles of synchrotron radiation losses.

PACS numbers: 12.20.-m, 52.27.Ep, 52.38.Ph

The lasers nowadays provide one of the most powerful sources of electromagnetic (EM) radiation under laboratory conditions and thus inspire the fast growing area of high field science aimed at the exploration of novel physical processes [1]. Lasers have already demonstrated the capability to generate light with the intensity of 2×10^{22}W/cm^2 [2] and projects to achieve 10^{26}W/cm^2 [3] are under way. Further intensity growth towards and above 10^{23}W/cm^2 will bring us to experimentally unexplored regimes. At such intensities the laser interaction with matter becomes strongly dissipative, due to efficient EM energy transformation into high energy gamma rays [1, 4]. These gamma-photons in the laser field may produce electron-positron pairs via the Breit-Wheeler process [5]. Then the pairs accelerated by the laser generate high energy gamma quanta and so on [6], and thus the conditions for the avalanche type discharge are produced at the intensity $\approx 10^{25}$ W/cm^2. The occurrence of such "showers" was foreseen by Heisenberg and Euler [7]. In Ref. [8] a conclusion is made that depletion of the laser energy on the electron-positron-gamma-ray plasma (EPGP) creation could limit attainable EM wave intensity and could prevent approaching the critical quantum electrodynamics (QED) field. This field [7, 9] is also called the Schwinger field, $E_S = m_e^2c^3/e\hbar$ corresponding to the intensity of $\approx 10^{29}$W/cm^2.

The particle-antiparticle pair creation by the Schwinger field cannot be described within the framework of perturbation theory and sheds light on the nonlinear QED properties of the vacuum [10]. Understanding the vacuum breakdown mechanisms is challenging for other nonlinear quantum field theories [11] and for astrophysics [12]. Reaching this field limit has been considered as one of the most intriguing scientific problems. Demonstration of the processes associated with the effects of nonlinear QED, such as vacuum polarization and vacuum electron-positron pair production, will be one of the main challenges for extreme high power laser physics [1, 13].

In the present paper we discuss the attainability of the Schwinger field with high power lasers. We compare the role of radiation dissipative effects in the motion of electrons (and positrons) produced via the Schwinger effect and show their dependence on the EM wave polarization.

Pair creation is determined by the Poincare invariants $\mathfrak{F} = (\mathbf{E}^2 - \mathbf{B}^2)/2$, $\mathfrak{G} = (\mathbf{E}\cdot\mathbf{B})$ and requires the first invariant $\mathfrak{F}$ be positive. This condition can be fulfilled in the vicinity of the antinodes of colliding EM waves, or/and in the configuration formed by several focused EM pulses, [15]. This EM configuration locally can be approximated by an oscillating TM mode with poloidal electric and toroidal magnetic fields. The magnetic field in spherical coordinates R, θ, ϕ is given by

$$\mathbf{B}(R,\theta) = \mathbf{e}_\phi \frac{a_0 \sin(\omega_0 t)}{(8\pi R)^{1/2}} J_{n+1/2}(k_0 R) L_n^l(\cos\theta), \qquad (1)$$

where $a_0 = eE_0/m_e c\omega_0$, $k_0 = \omega_0/c$, $J_\nu(x)$ and $L_n^l(x)$ are the Bessel function and associated Legendre polnomials. The electric field is equal to $\mathbf{E} = ik_0(\nabla\times\mathbf{B})$. In cylindrical coordinates r, ϕ, z the z-component of the electric field oscillates in vertical direction, $\sim a_0\cos(\omega_0 t)$, the ϕ-component of the magnetic field vanishes on the axis being linearly proportional to the radius, $\sim (a_0/8)k_0 r \sin(\omega_0 t)$, and the radial component of the electric field is relatively small, $\sim 0.1 a_0 k_0^2 r z\cos(\omega_0 t)$. The EM field and first Poincare invariant $\mathfrak{F}(r,z)$ are shown in Fig. 1. We see that the EM field is localized in a region of width less than the laser wavelength, $\lambda_0 = 2\pi/k_0$. The second invariant is equal to zero, $\mathfrak{G} = 0$.

Using expression for the probability of electron-positron pair creation [7, 9] and expanding $\mathfrak{F}(r,z)$ in the

---

*Also at Institute of Theoretical and Experimental Physics, Moscow 117218, Russia; Present address: University of California, Berkeley, CA 94720, USA

†Also at Prokhorov Institute of General Physics, Russian Academy of Sciences, Moscow 119991, Russia

vicinity of its maximum we find that the pairs are created in a small 4-volume near the electric field maximum with the characteristic size

$$\pi r_0^2 z_0 t_0 \approx \frac{5^{3/2}\lambda_0^4}{16\pi^5 c}\left(\frac{a_0}{a_S}\right)^2 . \qquad (2)$$

Here, we introduce $a_S = eE_S/m_e\omega_0 c = m_e c^2/\hbar\omega_0$. Integrating over the 4-volume the probability of the pair creation [16] we obtain the number of pairs produced per wave period, $(5^{3/2}/4\pi^3)a_0^4\exp(-\pi a_S/a_0)$, i. e. the first pairs can be observed for an one-micron wavelength laser intensity of the order of $2\times10^{27}\mathrm{W/cm}^2$, which corresponds to $a_0/a_S \approx 0.075$, i.e. a characteristic size, r_0, approximately equal to $0.04\lambda_0$.

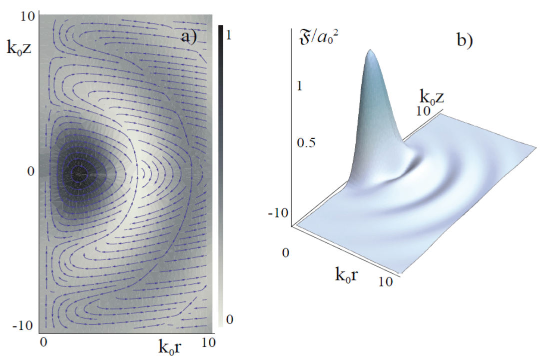

FIG. 1: a) The vector field shows r- and z-components of the poloidal electric field in the r,z plane for the TM mode. The color density shows the toroidal magnetic field distribution, $B_\phi(r,z)$. b) The first Poincare invariant $\mathfrak{F}(r,z)$.

In the region, where the magnetic field vanishes, the electron oscillates along the electric field. For an electron generated at small but finite radius $r_0 \ll \lambda_0$ the magnetic field bends its trajectory outwards. By solving the electron equations of motion linearized about the solution corresponding to ultrarelativistic electron oscillations in the z-direction, i.e. $a_0\omega_0 t \gg 1$, we can find the electron trajectories, which are described in terms of modified Bessel functions. The instability growth rate is approximately equal to half the EM field frequency, $\omega_0/2$, i. e. the electron remains in the close vicinity of the zero-magnetic field region leaving it along the z-direction.

The electron oscillating along the electric field emits the high frequency EM radiation with the power $\approx (2\pi r_e/3\lambda_0)\omega_e m_e c^2\gamma_e^2$ proportional to the square of electron energy. In order to find the angular distribution and frequency spectrum of the radiation in this case we should take into account its dependence on the retarded time: $t' = t - \mathbf{n}\cdot\mathbf{r}(t)/c$. Here $\mathbf{n}$ is the unit vector in the direction of observation and $\mathbf{r}(t)$ is the electron coordinate. Introducing the angle η between vectors $\mathbf{n}$ and $\mathbf{r}(t)$, $\mathbf{n}\cdot\mathbf{r}(t) = |\mathbf{r}(t)|\cos\eta$, we can find that in the direction of electron oscillations, $\eta = 0$, the radiation intensity vanishes. The maxima of the radiated power correspond to the angle η_m, for large γ_e, inversely proportional to the particle energy: $\eta_m \approx 1/2\gamma_e$.

The Fourier components of the 4-vector potential of the EM field according to Ref. [14] are

$$A^\mu(\omega) = \frac{e}{R}\int_{-\infty}^{+\infty}\frac{u^\mu}{c}\exp\left\{i\omega\left[t - \frac{1}{c}\mathbf{n}\cdot\mathbf{r}(t)\right]\right\}dt, \qquad (3)$$

where $u^\mu = p^\mu/m_e\gamma_e$ is the four-velocity. $\mathbf{r}(t) = \mathbf{e}_z(c/\omega_0)\mathrm{Arcsin}\left[\beta_m\sin(\omega_0 t)\right]$, and $\beta_m = a_0(1+a_0^2)^{-1/2}$. Expanding the phase in expression (3), $\Phi(t) = \omega\left\{t - (\cos\eta/\omega_0)\,\mathrm{Arcsin}\left[\beta_m\sin(\omega_0 t)\right]\right\}$, over small parameters, $\gamma_{e,m}^{-1}$ and $\omega_0 t$, for $\eta = \eta_m \approx 1/2\gamma_{e,m}$, we obtain

$$\Phi(t) \approx \omega\left[(1-\beta_m\cos\eta)\,t + \frac{\beta_m\cos\eta}{6\omega_0\gamma_{e,m}^2}(\omega_0 t)^3\right]. \qquad (4)$$

Using the Airy integral, we can find the y-component of the 4-vector potential of the EM field (3) and the radiation power density. Since $\eta \sim 1/\gamma_{e,m} \ll 1$, and thus $\cos\eta - 1 \sim 1/2\gamma_{e.m}^2$, the maximum frequency of the radiation emitted by the linearly oscillating electron is $\omega_m \approx 0.21\omega_0\gamma_{e,m}^2$.

To take into account the radiation friction we use equation of motion of a radiating electron [14]. We can estimate the regime where the radiation friction can become relatively large by comparing the energy losses with the maximal energy gain of an electron accelerated by the electric field, $\dot{\mathcal{E}}^{(+)} \approx \omega_0\, m_e c^2 a_0$, i.e. $\omega_0\, m_e c^2 a_0 = \varepsilon_{rad}\omega_0 m_e c^2\gamma_e^2$, where $\varepsilon_{rad} = 4\pi r_e/3\lambda_0$, with $r_e = e^2/m_e c^2$. As is apparent, although an electron moving along the oscillating electric field loses energy, radiation friction effects may become important only at $a_0 = 2\varepsilon_{rad}^{-1}$, i.e. at the electric field $E_0 = 3m_e^2c^4/e^3$, which is of the order of the critical electric field of classical electrodynamics (see also Ref. [16]). This is 137 times larger than the field E_S.

In QED the charged particle interaction with EM fields is determined by relativistically and gauge invariant parameters [18] $\chi_e = [(F_{\mu\nu}p_\nu)^2]^{1/2}/m_e c E_S$. The parameter, χ_e, characterizes the probability of the gamma-photon emission by the electron with Lorentz factor γ_e. It is of the order of the ratio E/E_S in the electron rest frame of reference. Another parameter, $\chi_\gamma = [(F_{\mu\nu}\hbar k_\nu)^2]^{1/2}/m_e c E_S$, is similar to χ_e with the photon 4-momentum, $\hbar k_\mu$, instead of the electron 4-momentum, p_μ. It characterizes the probability of the electron-positron pair creation due to the collision between the high energy photon and EM field. QED effects come into play when the energy of a photon emitted by an electron becomes comparable to the electron kinetic energy, i.e., for $\hbar\omega_m = m_e c^2\gamma_e$. In a linearly polarized oscillating electric field the maximum frequency of emitted photons, ω_m, is proportional γ_0^2, and, therefore, quantum effects should be incorporated into the theoretical description at the electron energy corresponding to the gamma-factor $\gamma_Q^L = m_e c^2/0.21\,\hbar\omega_0$, which is above the Schwinger limit. We see that in the case of electron motion in a linearly

polarized oscillating electric field neither radiation friction nor quantum recoil effects are important.

Reaching the threshold of an avalanche type discharge with EPGP generation discussed in Refs. [6, 8] requires high enough values of the parameters χ_e and χ_γ defined above because for $\chi_\gamma \ll 1$ the rate of the pair creation is exponentially small [19], $W(\chi_\gamma) \approx \alpha\left(m_e^2 c^4/\hbar^2\omega_\gamma\right)\chi_\gamma \exp\left(-8/3\chi_\gamma\right)$. In the limit $\chi_\gamma \gg 1$ the pair creation rate is given by $W(\chi_\gamma) \approx \alpha\left(m_e^2 c^4/\hbar^2\omega_\gamma\right)(\chi_\gamma)^{2/3}$ (for details see Ref. [18]). Here $\hbar\omega_\gamma$ is the energy of the photon which creates an electron-positron pair.

Since for $\gamma_e \geq \gamma_Q$ the photon is emitted by the electron (positron) in a narrow angle almost parallel to the electron momentum with the energy of the order of the electron energy, the parameters χ_e and χ_γ are approximately equal to each other. The parameter χ_e can be expressed via the electric and magnetic field as (see Ref. [18])

$$\chi_e^2 = \left(\gamma_e \frac{\mathbf{E}}{E_S} + \frac{\mathbf{p}\times\mathbf{B}}{m_e c E_S}\right)^2 - \left(\frac{\mathbf{p}\cdot\mathbf{E}}{m_e c E_S}\right)^2 . \qquad (5)$$

In order to find the threshold for the avalanche development we need to estimate the QED parameter χ_e. The condition for avalanche development corresponding to the parameter χ_e should become of the order of unity within one tenth of the EM field period (e.g. see Ref. [8]). Due to the trajectory bending by the magnetic field the electron transverse momentum changes as $p_\perp \approx (a_0/16)k_0 r_0(\omega_0 t)^2$, where $k_0 r_0 = (2.5a_0/\pi a_s)^{1/2}$, Eq. (2). Assuming $\omega_0 t$ to be equal to 0.1 π, we obtain from Eq. (5) that χ_e becomes of the order of unity, i.e. the avalanche can start, at $a_0/a_S \approx 0.105$, which corresponds to the laser intensity $4\times10^{27}\mathrm{W/cm^2}$. The radiation losses in this limit can be described as the synchrotron losses of an electron with the energy $\approx m_e c^2$ moving in the magnetic field $a_0(k_0 r_0)/8$. Using formulae for synchrotron radiation [14], it is easy to show that they do not become significant until $a_0 \approx 5\times10^4$. At that limit the Schwinger mechanism provides approximately 5×10^5 pairs per one-period.

In the case of two colliding circularly polarized EM waves the resulting electric field rotates with frequency ω_0 being constant in magnitude. The power emitted by the electron is $\approx \varepsilon_{rad}\omega_0 m_e c^2\gamma_e^4$. This is a factor of γ_e^2 larger than in the case of linear polarization. The properties of radiation emitted by rotating electron are well known from the theory of synchrotron radiation [14, 16] and from Ref. [17]. In the limit $\gamma_e \gg 1$ the emitted power is proportional to the fourth power of the electron energy. The radiation is directed almost along the electron momentum being localized within the angle inversely proportional to the electron energy: $\delta\eta \approx 1/\gamma_e$. The frequency spectrum given by the well known expression [14] has a maximum frequency, $\omega_m = 0.29\omega_0\gamma_e^3$, proportional to the cube of the electron energy. This is a factor of γ_e larger than in the case of linear polarization. For the electron rotating in the circularly polarized colliding EM waves the emitted power becomes equal to the maximal energy gain at the field amplitude $a_0 = a_{rad} = \varepsilon_{rad}^{-1/3}$. For the laser wavelength $\lambda_0 = 0.8\,\mu$m $\varepsilon_{rad} = 2.2\times10^{-8}$. The normalized amplitude a_{rad} is ≈ 400 corresponding to the laser intensity $I_{rad} = 4.5\times10^{23}\mathrm{W\ /\ cm^2}$.

We represent the electric field and the electron momentum in the complex form: $E = E_y + iE_z = E_0\exp(-i\omega_0 t)$ and $p = p_y + ip_z = p_\perp\exp(-i(\omega_0 t - \varphi))$, where φ is the phase equal to the angle between the electric field vector and the electron momentum. In the stationary regime, when the electron rotates with constant energy, the equations for the electron energy, $\gamma_e = [1 + (p_\perp/m_e c)^2]^{1/2}$, and for the phase φ have the form

$$a_0^2 = \left(\gamma_e^2 - 1\right)\left(1 + \varepsilon_{rad}^2\gamma_e^6\right) \text{ and } \tan\varphi = -\frac{1}{\varepsilon_{rad}\gamma_e^3}. \quad (6)$$

In the limit of weak radiation damping, $a_0 \ll \varepsilon_{rad}^{-1/3}$, the absolute value of the electron momentum is proportional to the electric field magnitude, $p_\perp = m_e c a_0$, while in the regime of dominant radiation damping effects, i.e. at $a_0 \gg \varepsilon_{rad}^{-1/3}$, it is given by $p_\perp = m_e c\,(a_0/\varepsilon_{rad})^{1/4}$. For the momentum dependence given by this expression the power radiated by an electron is $P_{\gamma,C} = \omega_0 m_e c^2 a_0$, i.e. the energy obtained from the driving electromagnetic wave is completely re-radiated in the form of high energy gamma rays. At $a_0 \approx \varepsilon_{rad}^{-1/3}$ we have for the gamma photon energy $\hbar\omega_\gamma = 0.29\hbar\omega_0 a_{rad}^3 \approx 0.45\hbar\omega_0\left(mc^3/e^2\right)$. For example, if $\lambda_0 \approx 0.8\,\mu$m and $a_0 \approx 400$ the circularly polarized laser pulse of intensity $I_{rad} = 4.5\times10^{23}\,W/cm^2$ generates a burst of gamma photons of energy about 20 MeV with the duration determined either by the laser pulse duration or by the decay time of the laser pulse in a plasma.

Since in the case of circular polarization ω_m is proportional to the cube of electron gamma-factor quantum effects should be incorporated into the theoretical description at $\gamma_e \approx \gamma_Q^C = (m_e c^2/0.29\,\hbar\omega_0)^{1/2} \approx 1300$. For $\gamma_e = a_0$ this limit is reached at the intensity of $\approx 3.4\times10^{24}\mathrm{W/cm^2}$. The electron motion should be described within the framework of quantum mechanics. These effects change the radiative loss function (see Ref. [18]). In the quantum regime, it is necessary to take into account not only radiative damping effects but also recoil momentum effects, which change the direction of motion of the electron because the outgoing photon carries away the momentum $\hbar k_m = \hbar\omega_m/c$.

In the regime when the radiation friction effects are important, i.e. when $a_0 \gg \varepsilon_{rad}^{-1/3}$, the angle φ between the electron momentum and the electric field is small being equal to $\left(\varepsilon_{rad}a_0^3\right)^{-1/4}$, i. e. the electron moves almost in the electric field direction. The electron momentum is given by $p_\perp = m_e c\,(a_0/\varepsilon_{rad})^{1/4}$. This yields an estimation $\chi_e \approx (a_0/a_S^2\varepsilon_{rad})^{1/2}$. This becomes greater than unity for $a_0 > \varepsilon_{rad}a_S^2 \approx 5.5\times10^3$, which corresponds to the laser intensity equal to $6\times10^{25}\mathrm{W/cm^2}$. In Ref. [8]

an avalanche threshold intensity several times lower has been found neglecting the effects of the radiation friction force (see also [20]). However, the radiation friction time is of the order of $t_{rad} = 1/\omega_0 \left(\varepsilon_{rad} a_0^3\right)^{1/2}$, which for $a_0 \approx 5.5 \times 10^3$ is approximately one tenth of the laser period. Hence the radiation friction effects do not prevent the EPGP cascade development for circularly polarized colliding waves. Such a prolific electron-positron pair and gamma ray creation [6] should result in the EPGP generation.

While creating and then accelerating the electron-positron pairs the laser pulse generates an electric current and EM field. The electric field induced inside the EPGP cloud with a size of the order of the laser wavelength, λ_0 can be estimated to be $E_{pol} = 2\pi e(n_+ + n_-)\lambda_0$. Here $n_+ \approx n_-$ are the electron and positron density, respectively. Coherent scattering of the laser pulse away from the focus region occurs when the polarization electric field becomes equal to the laser electric field. This yields for the electron and positron density $n_+ \approx n_- = E/4\pi e\lambda_0$. The particle number per λ_0^3 volume is about $a_0\lambda_0/r_e$. This is a factor a_0 smaller than required for the laser energy depletion.

In conclusion, the high enough laser intensity pulse with arbitrary polarization plus high enough density of seed electrons, e.g. generated in the laser interaction with solid targets can provide necessary and sufficient conditions for the avalanche development, [6]. Instead, in vacuum, when the seed electrons(positrons) are created via the Schwinger mechanism, we see a fundamental difference between the circularly and linearly polarized waves. In the case of the circularly polarized EM wave the electron radiation is strong and the threshold for the avalanche is low enough for avalanche starting at the laser intensity well below the Schwinger limit. Since, as noted in Ref. [6], the electron-positron avalanche parameters are insensitive to the seed electrons (positrons), the parameters of the Schwinger created pairs become hidden and can hardly be revealed. Contrary to this, in the linearly polarized EM wave is more favorable for the realization and reaching of "pure" Schwinger electron-positron pair creation. An electron moving along the electric field with velocity and acceleration parallel to the field emits much fewer photons with substantially lower energy neither experiencing the radiation friction nor quantum recoil effects. We see an analogy of these cases with circular and linear electron accelerators with the corresponding constraining and reduced roles of synchrotron radiation losses. The electron-positron pair creation in the Breit-Wheeler type process is also suppressed because the key parameters χ_e and χ_γ dependence on the electron and photon momentum, in the laser field with the same intensity,is much weaker.

We thank S. G. Bochkarev, V. Yu. Bychenkov, P. Chen, E. Esarey, A. M. Fedotov, V. F. Frolov, D. Habs, M. Kando, K. Kondo, G. Korn, N. B. Narozhny, W. Rozmus, H. Ruhl, and A. I. Zelnikov for discussions. We acknowledge support of this work from MEXT of Japan, Grant-in-Aid for Scientific Research, No. 20244065 and from NSF through the Frontiers in Optical and Coherent Ultrafast Science Center at the University of Michigan.

[1] G. A. Mourou et al., Rev. Mod. Phys. **78**, 309 (2006); M. Marklund and P. Shukla, Rev. Mod. Phys. **78**, 591 (2006); Y. I. Salamin et al., Phys. Rep. **427**, 41 (2006).
[2] V. Yanovsky et al., Opt. Express **16**, 2109 (2008).
[3] M. Dunne, Nature Phys. **2**, 2 (2006); www.eli-laser.eu
[4] Ya. B. Zel'dovich, Sov. Phys. Usp. **18**, 79 (1975); A. Zhidkov et al., Phys. Rev. Lett. **88**, 185002 (2002); S. V. Bulanov et al., Plasma Phys. Rep. **30**, 196 (2004).
[5] G. Breit and J. A. Wheeler, Phys. Rev. **46**, 1087 (1934); C. Bamber et al., Phys. Rev. D **60**, 092004 (1999); H. Hu et al., Phys. Rev. Lett. **105**, 080401 (2010).
[6] A. R. Bell and J. G. Kirk, Phys. Rev. Lett. **101**, 200403 (2008); J. G. Kirk et al., Plasma Phys. Contr. Fusion **51**, 085008 (2009).
[7] W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936).
[8] A. M. Fedotov et al., Phys. Rev. Lett. **105**, 080402 (2010).
[9] F. Sauter, Z. Phys. **69**, 742 (1931); J. Schwinger, Phys. Rev. **82**, 664 (1951).
[10] W. Dittrich and H. Gies, Probing the Quantum Vacuum (Springer. Berlin. 2000).
[11] A. Casher et al., Phys. Rev. D **20**, 179 (1979); Y. Kluger et al., Phys. Rev. D **58**, 125015 (1998); C. K. Dumlu and G. V. Dunne, Phys. Rev. Lett. **104**, 250402 (2010).
[12] R. Ruffini et al., Phys. Rep. **487**, 1 (2010).
[13] A. Ringwald, Phys. Lett. B **510**, 107 (2001); V. S. Popov, Phys. Lett. A **298**, 83 (2002); N. B. Narozhny et al., Phys. Lett. A **330**, 1 (2004); S. S. Bulanov et al., JETP **102**, 9 (2006); A. Di Piazza et al., Phys. Rev. Lett. **103**, 170403 (2009); R. Schutzhold, Adv. Sci. Lett. **2**, 121 (2009); G. V. Dunne et al., Phys. Rev. D **80**, 111301(R) (2009).
[14] L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields (Pergamon Press. Oxford. 1975); V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, Quantum Electrodynamics (Pergamon. New York. 1982).
[15] S. S. Bulanov et al., Phys. Rev. Lett. **104**, 220404 (2010).
[16] J. Schwinger, Phys. Rev **75**, 1912 (1949).
[17] E. Sarachik and G. Schappert, Phys. Rev. D **1**, 2738 (1970); E. Esarey et al., Phys. Rev. E **48**, 3003 (1993).
[18] V. I. Ritus, Tr. Fiz. Inst. Akad. Nauk SSSR **111**, 6 (1979).
[19] H. R. Reiss, J. Math. Phys. **3**, 59 (1962); A. I. Nikishov and V. I. Ritus, Sov. Phys. Usp. **13**, 303 (1970).
[20] We note here that using the Landau-Lifshitz approximation for the radiation friction force in the limit $a_0 > a_{rad}$ leads to the losses scaling as $\sim \varepsilon_{rad} a_0^2 \gamma_e^2$ instead $\sim \varepsilon_{rad}\gamma_e^4$ and to underestimation of the laser intensity at which the parameter χ_e becomes equal to unity.